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N-PORT THEORY APPLIED TO BACKSCATTER POLARIMETRY AND DEPOLARIZATION IN FOLIAGE PENETRATION

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ABSTRACT

This tutorial paper on polarimetric definitions uses N-port network theory to prove results that are frequently stated without proof in the literature, including the symmetry of the polarization scattering matrix in backscattering, the equation for received voltage, and the equations defining Stokes vectors and backscatter Mueller matrices. Appropriate N-port networks are defined for a single-polarized antenna, a dual-polarized antenna, and a backscattering target. An important result is demonstrated: reciprocity (symmetry of the polarization scattering matrix) is meaningful only in the context of the monostatic convention, for which the coordinate system is the same for both transmit and receive. This, in turn, implies a change in the handedness of the coordinate system, so that scattered fields and Stokes vectors are expressed in opposite handedness from incident fields and Stokes vectors.

These results are used as tools in the derivation of a new technique for measuring depolarization in foliage penetration (FOPEN). Four polarimetric active radar calibrators (PARCs) are used with an algorithm to completely measure the Mueller matrices describing downward-going and upward-going foliage penetration.

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1. INTRODUCTION AND CONCLUSIONS

This is a tutorial paper on polarimetric definitions (scattering matrices, Stokes vectors, and backscatter Mueller matrices). The paper contains a number of results which have proved useful to the writer, and also introduces the notion of coordinate-system handedness, which, it is felt, is not adequately treated in the literature.

These results are used to develop a new technique for measuring depolarization in foliage penetration (FOPEN): four polarimetric active radar calibrators (PARCs) are used with an algorithm to completely measure the Mueller matrices describing depolarization during downward-going and upward-going foliage penetration.

The major results derived in this paper include the following:

1. The polarization scattering matrix for backscatter is symmetric <u>only</u> if the incident field and backscattered field are expressed in coordinate systems that have opposite handedness. For example, if the transmit field is expressed in a right-handed system, the backscattered field must be expressed in a left-handed system. If one transforms the backscattered field so that it also is in a right-handed system, then the scattering matrix is <u>not</u> symmetric.

2. The voltage measured by a receive antenna can be written as a simple dot product $\underline{r}^T \tilde{\underline{e}}$ between the receive antenna complex height $\underline{r}$ and the backscattered field $\tilde{\underline{e}}$, where $\tilde{\underline{e}}$ is expressed in a left-handed coordinate system. This is <u>not</u> an inner product (e.g. $\underline{r}^\dagger \tilde{\underline{e}}$), where the superscript dagger denotes conjugate transpose, but rather a simple dot product.

3. The receive distortion matrix B of a dual-polarized antenna combined with an orthomode transducer is equal to the transmit distortion matrix C. This result follows from reciprocity, and is therefore not generally true for a complete receiver, which may include nonreciprocal elements such as circulators.

4. The equation for the receive antenna's measured power can be written in terms of the Stokes vectors of the receive antenna and the backscattered field. The backscattered-field Stokes vector must be expressed in a left-handed coordinate system.

5. The backscatter Mueller matrix (also known as the Stokes reflection matrix) is symmetric only if the incident and scattered Stokes vectors are expressed in coordinate systems with opposite handedness.

6. The transmission Mueller matrix (for example, the matrix describing depolarization through foliage) is <u>not</u> symmetric, and therefore has sixteen and not nine independent elements

Section 2 uses N-port network theory to derive several simple relationships between complex two-vectors and polarization scattering matrices. Section 3 applies the results from Section 2 to demonstrate relationships between Stokes vectors and backscatter Mueller matrices. Section 4 extends the backscatter results to forward scatter, and then uses the results as tools to develop a technique for measuring depolarization in foliage penetration.

2. N-PORT NETWORK THEORY APPLIED TO BACKSCATTERING AND ANTENNAS

Engineering progress can frequently be made by exploiting analogies with results developed in similar fields. Such an analogy is proposed here, between radar antennas and scattering objects on the one hand, and a branch of linear systems theory called N-port network theory on the other hand. In this section we will model a backscattering object as a two-port network, a single-polarized antenna as a three-port network, and a dual-polarized antenna as a four-port network. In order to do this, some basic results from N-port network theory ([2], e.g.) are first recalled.

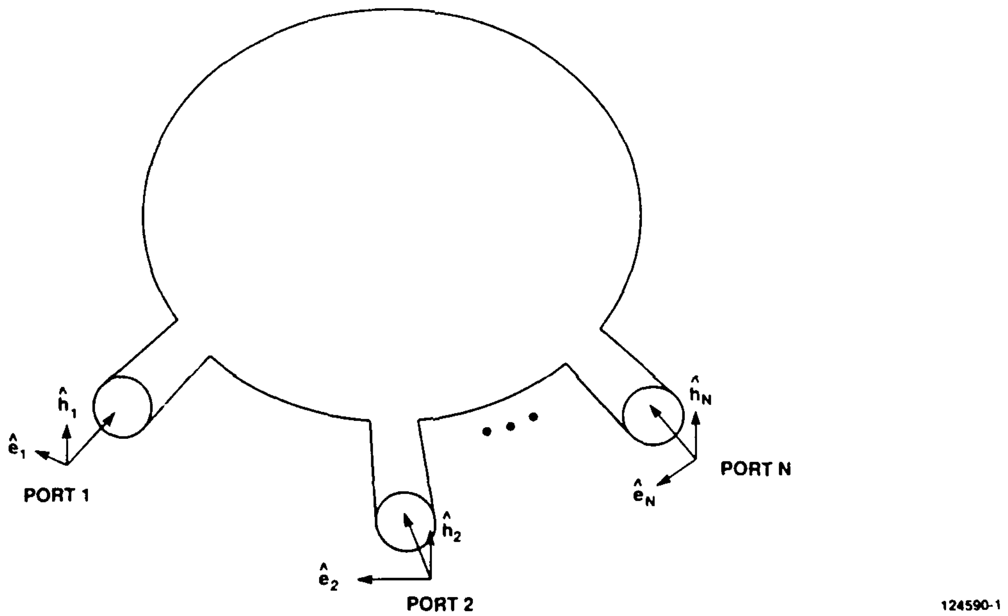

Figure 2-1. N-port network.

An N-port network (see Figure 2-1) can be characterized by a scattering matrix, which relates backscattered to incident waves. At port m, the total electric field can be expressed as $\vec{E}_m = (a_m + b_m)\hat{e}_m$, while the total magnetic field is $\vec{H}_m = (a_m - b_m)\hat{h}_m$, where $\hat{e}_m$ and $\hat{h}_m$ are the unit vectors for the electric and magnetic fields at the m-th port, a_m is the complex coefficient of the inward wave at the m-th port, and b_m is the complex coefficient of the outward wave at the m-th port. Note that both inward and outward waves are defined with respect to the <u>same</u> coordinate system $(\hat{e}_m, \hat{h}_m)$. The scattering matrix S relates these coefficients as follows:

$$\underline{b} = S\underline{a} \tag{2.1}$$

$$\text{where } \underline{b} = \begin{bmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_N \end{bmatrix} \text{ and } \underline{a} = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_N \end{bmatrix} \tag{2.2}$$

and where the element S_{ij} is the amplitude of an outward wave at port i resulting from a unit inward wave at port j, with all other ports "matched" (i.e. with no other inward waves). In this expression $\underline{a}$ and $\underline{b}$ are, in general, complex N-vectors.

An important result from N-port network theory is that when an N-port network is passive and reciprocal, its scattering matrix S is symmetric, i.e., $S_{ij} = S_{ji}$. This result will be used extensively in this paper.

2.1 The Backscatterer as a Two-port Network

N-port network theory can be used to model a backscattering object as a two-port network. Define Ports 1 and 2 as corresponding to horizontally and vertically polarized electric fields, respectively; then the unit vectors $\hat{e}_1$ and $\hat{e}_2$ correspond to $\hat{x}_t$ and $\hat{y}_t$, respectively, in Figure 2-2. The particular choice for $\hat{e}_1$ and $\hat{e}_2$ is not important; what is important is that the <u>same</u> coordinate system is used for both inward and outward waves (i.e. for the incident and backscattered fields). Since the propagation vectors for the incident and backscattered fields are in opposite directions, this means that the incident and scattered coordinate systems must have opposite handedness. We arbitrarily choose a right-handed triplet $(\hat{x}_t, \hat{y}_t, \hat{z}_t)$ for the transmit coordinate system and a left-handed triplet $(\hat{x}_s, \hat{y}_s, \hat{z}_s)$ for the backscattered coordinate system, as shown in Figure 2-2.

With these definitions, the polarization scattering matrix A for a backscattering object can be viewed as the N-port scattering matrix S for the case $N = 2$. We will now find analogies for each of the parts ($\underline{a}, S$, and $\underline{b}$) of Equation 2.1:

1. The vector $\underline{a}$ of incoming waves corresponds to the incident field two-vector $\underline{t}$, which we use to denote the field transmitted by the transmit antenna. Note that $\underline{t}$ is expressed in a right-handed coordinate system.

2. The vector $\underline{b}$ of the outgoing waves corresponds to the backscattered wave $\underline{\tilde{e}}$, (the tilde denotes that $\underline{\tilde{e}}$ is expressed in a left-handed coordinate system).

3. Finally, the N-port scattering matrix S corresponds to the polarization scattering matrix A for the backscattering target.

Combining these substitutions with the original N-port equation (Equation 2.1), we get:

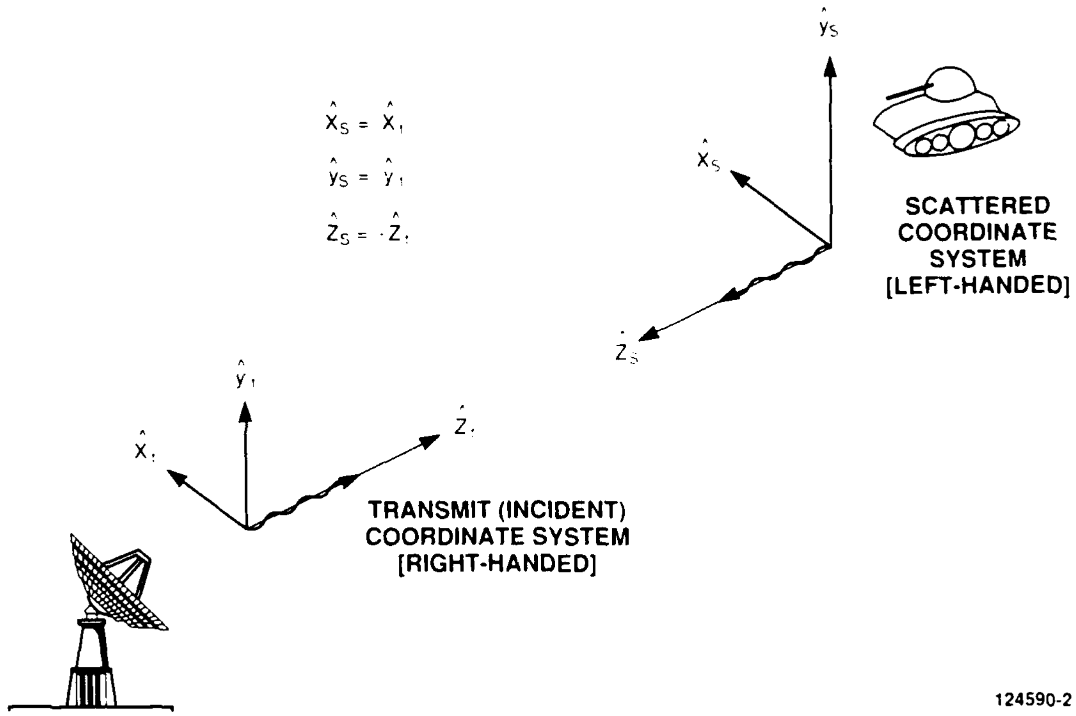

Figure 2-2. Geometry for a scatterer as a two-port network.

$$\underline{\tilde{e}} = A\underline{t} \tag{2.3}$$

As an example of the above result, consider a left-circular wave (expressed in a right-handed coordinate system) which is assumed to be incident on a sphere. The incident wave can be expressed as $\underline{t} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$, and the scattering matrix of a sphere is $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; thus the scattered field is $\underline{\tilde{e}} = A\underline{t} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$. Physically, we know that the scattered field must be right-circular polarized. Thus, the same complex two-vector $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$ refers to left-circular in the right-handed coordinate system and right-circular in the left-handed coordinate system.

2.2 The Single-polarized Receive Antenna as a Three-port Network

N-port network theory can also be applied to modelling an antenna. In this section, we will model a single-polarized antenna as a three-port network. Define Ports 1 and 2 as corresponding to the horizontally and vertically polarized electric fields, respectively, transmitted or received by the antenna. Also, define Port 3 as the waveguide input/output at which the antenna's complex voltage is defined.

Assume that the antenna in question is not fitted out with an orthomode transducer. (If it were, it would have to be modelled as a four-port network; this case will be treated in Section 2.3.)

Let us assume that the the unit vectors $\hat{e}_1$ and $\hat{e}_2$ correspond to $\hat{x}_t$ and $\hat{y}_t$, respectively, in Figure 2-3. This means that transmitted waves are expressed in a right-handed coordinate system and received waves are expressed in a left-handed coordinate system.

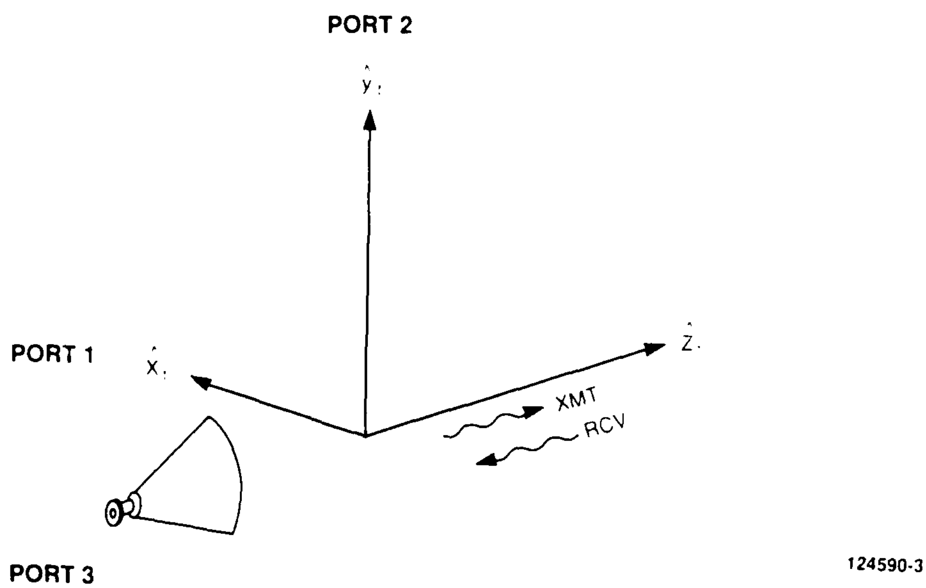

Figure 2-3. Geometry for a single-polarized antenna as a three-port network.

Consider the antenna operating as a transmitter. Assume there is a unit incoming wave at Port 3 (i.e., the signal to be transmitted coming into the antenna waveguide input/output). Assume also that there are no waves incident on the antenna from the rest of the world, so that the incoming waves at Ports 1 and 2 are zero. The $\underline{a}$-vector (the complex three-vector describing the coefficients of the incoming waves to the three ports of the antenna) can therefore be expressed as $[\ 0 \quad 0 \quad 1\ ]^T$, so that (according to Equation 2.1) the resulting $\underline{b}$-vector (the complex three-vector of outgoing coefficients) is the <u>last</u> column of the S matrix:

$$\underline{b} \;=\; S\underline{a} = \begin{bmatrix} S_{13} \\ S_{23} \\ S_{33} \end{bmatrix} \tag{2.4}$$

But the first two elements of $\underline{b}$ are the horizontal and vertical components of the field that this antenna transmits. Call these g_1 and g_2. (The field $\underline{g}$ is a dummy parameter; it is set to $\underline{t}$ if the antenna is to be used as a transmit antenna, and to $\underline{r}$ if it is to be used as a receive antenna.) Thus, $S_{13} = g_1$ and $S_{23} = g_2$. Because the S matrix is symmetric, we also have $S_{31} = g_1$ and $S_{32} = g_2$.

Now consider the operation of the antenna as a receiver. Here the analogy with the N-port network is as follows: the role of the incoming wave is played by the electric field incident on the antenna from the outside world, while the role of the outgoing wave is played by the signal sent down the waveguide from the antenna to the receiver.

Assume that an incident wave with horizontal component e_1 and vertical component e_2 arrives at Ports 1 and 2, and that no signals are coming into Port 3 (the antenna waveguide input/output) from the receiver. Thus the incoming wave vector to be used with the three-port network is $\underline{a} = [\ e_1 \quad e_2 \quad 0\ ]^T$. Let us examine the value of the outgoing wave in Port 3, i.e., the antenna received voltage. This is the third component of the $\underline{b}$-vector, which is equal to

$$V \quad = \quad b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 = g_1e_1 + g_2e_2 \tag{2.5}$$

This result can then be applied to the receive antenna (for which $\underline{g}$ is taken to be $\underline{r}$):

$$V \quad = \quad \underline{r}^T\underline{\tilde{e}} \tag{2.6}$$

Note that this is <u>not</u> an inner product; i.e. neither $\underline{r}$ nor $\underline{\tilde{e}}$ is to be conjugated.

2.3 The Dual-polarized Antenna as a Four-port Network

An orthomode transducer is a microwave device with an input/output that can support electric fields in either or both of two orthogonal modes (these modes are typically referred to as horizontally and vertically polarized), as well as two waveguide connections. A dual-polarized antenna is just an orthomode transducer mounted onto a feed/aperture structure capable of supporting both horizontal and vertical polarizations. Since we count one port for each waveguide connection and one port each for the horizontal and vertical components of the radiated electric field, the dual-polarized antenna can be modelled as a four-port network.

The analysis of this four-port network will closely parallel that in Section 2.2. Define Ports 1 and 2, respectively, as corresponding to the horizontally and vertically polarized electric fields transmitted or received by the antenna. Define Ports 3 and 4, respectively, as the horizontally and vertically polarized output of the orthomode transducer.

Let us assume that the the unit vectors $\hat{e}_1$ and $\hat{e}_2$ correspond to $\hat{x}_t$ and $\hat{y}_t$, respectively, in Figure 2-4. This means that transmitted waves are expressed in a right-handed coordinate system and received waves are expressed in a left-handed coordinate system.

Consider the antenna operating as a transmitter. In this mode, the two waveguide connections (horizontal and vertical) of the orthomode transducer are used as inputs. Although signals can

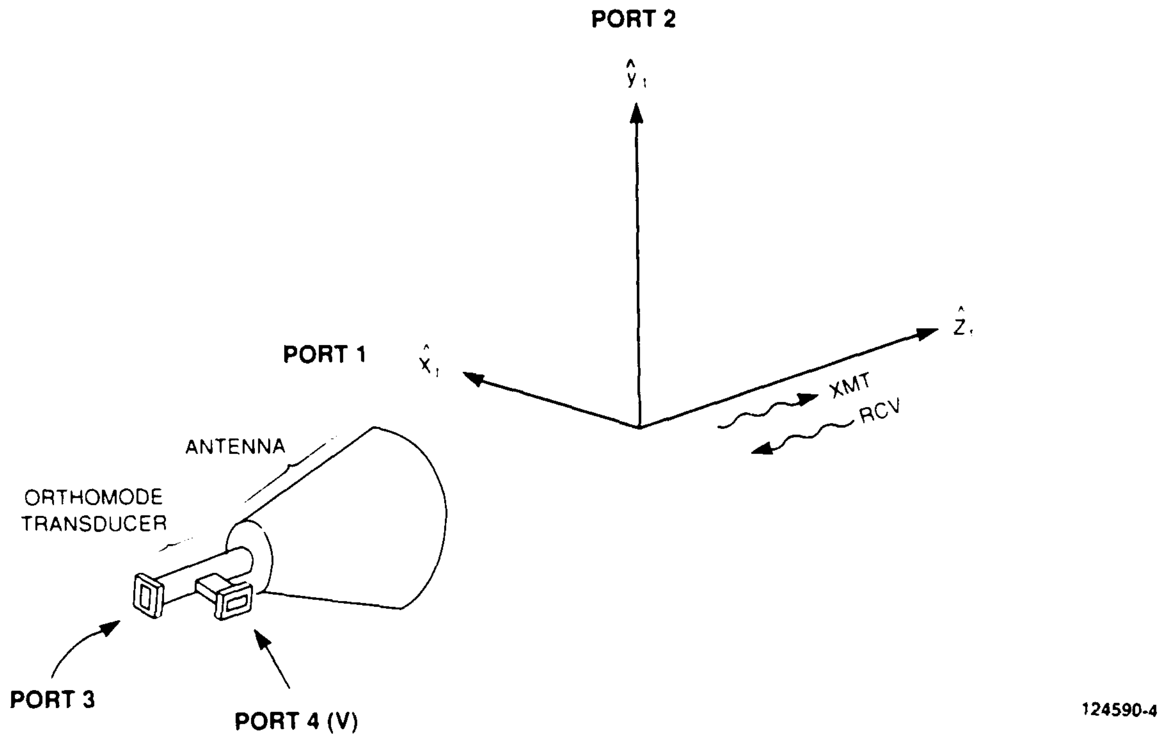

Figure 2-4. Geometry for a dual-polarized antenna as a four-port network.

be present in both waveguides at the same time, we will analyze the cases for which one one or the other waveguide input is used, and then appeal to linear superposition to find the field radiated when both waveguide inputs are used simultaneously.

First we assume that only the horizontal waveguide input is used.

Assume there is a unit incoming wave in Port 3 (i.e., assume that the signal to be transmitted is coming into the orthomode transducer horizontal input). Assume also that there are no waves incident on the antenna from the rest of the world, so that the incoming waves in Ports 1, 2, and 4 are zero. The $\underline{a}$-vector (the complex four-vector describing the coefficients of the incoming waves to the four ports of the antenna) is thus $[\ 0 \quad 0 \quad 1 \quad 0\ ]^T$, so that the resulting $\underline{b}$-vector (the complex four-vector of outgoing coefficients) is (by Equation 2.1) equal to the third column of the S matrix:

$$\underline{b} \quad = \quad S\underline{a} = \begin{bmatrix} S_{13} \\ S_{23} \\ S_{33} \\ S_{43} \end{bmatrix} \tag{2.7}$$

But the first two elements of $\underline{b}$ are the Port 1 and Port 2 outputs, i.e., the horizontal and vertical components of the field the antenna transmits when the nominal transmit polarization is horizontal. Call these two elements C_{11} and C_{21}, using the notation of [4]. The electric field actually transmitted when we input only to the horizontal waveguide is $\underline{C}_1 = [\ C_{11} \quad C_{21}\ ]^T$. Thus $S_{13} = C_{11}$ and $S_{23} = C_{21}$.

Next, we assume that only the vertical waveguide input is used.

The analysis is similar to that used for the horizontal waveguide. In this case, only the vertical input of the orthomode transducer is stimulated, so the $\underline{a}$-vector is equal to $[\ 0 \quad 0 \quad 0 \quad 1\ ]^T$. The resulting $\underline{b}$-vector is (by Equation 2.1) equal to the fourth column of the S matrix:

$$\underline{b} \;=\; S\underline{a} = \begin{bmatrix} S_{14} \\ S_{24} \\ S_{34} \\ S_{44} \end{bmatrix} \tag{2.8}$$

Thus, using the notation of [4], when we input to the vertical waveguide (i.e. into Port 4), the actual transmitted field is $\underline{C}_2 = [\ C_{12} \quad C_{22}\ ]^T$; since these two coefficients are equivalent to the first two elements of the fourth column of S, we have $S_{14} = C_{12}$ and $S_{24} = C_{22}$.

What has been shown so far is that the upper-right two-by-two submatrix of the four-by-four scattering matrix is the transmit distortion matrix $C = [\underline{C}_1 \underline{C}_2]$ defined in [4].

Now consider the operation of the antenna as a receiver. Assume that an incident wave with horizontal component e_1 and vertical component e_2 arrives at Ports 1 and 2 respectively, and that no signals are coming into Ports 3 and 4 (the orthomode transducer waveguide ports) from the receiver. Thus the incoming wave vector to be used with the four-port network analogy is $\underline{a} = [\ e_1 \quad e_2 \quad 0 \quad 0\ ]^T$. Let us examine the value of the outgoing wave in Port 3, i.e., the antenna output voltage in the horizontal orthomode transducer port. This is the third component of the $\underline{b}$-vector, which, using the symmetry of the S matrix, is given by:

$$\begin{aligned} V = b_3 \;&=\; S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4 \\ &=\; S_{13}a_1 + S_{23}a_2 + S_{33}a_3 + S_{43}a_4 \\ &=\; C_{11}e_1 + C_{21}e_2 \\ &=\; \underline{C}_1^T \underline{\tilde{e}} \end{aligned} \tag{2.9}$$

Similarly, the output voltage in Port 4 (vertical) is $V = b_4 = \underline{C}_2^T \underline{\tilde{e}}$.

The complex numbers b_3 and b_4 are the horizontal and vertical components of the distorted receive field (where the term "distorted" is the term used in [4] to refer to a measurement which differs from its nominal value). If b_3 and b_4 are arranged as a complex two-vector, which we call $\underline{\tilde{e}}'$, then

$$\underline{\tilde{e}}' \;=\; \begin{bmatrix} b_3 \\ b_4 \end{bmatrix} = C^T \underline{\tilde{e}} \tag{2.10}$$

But the receive distortion matrix B is defined in [4] by the equation $\underline{\tilde{e}}' = \begin{bmatrix} b_3 \\ b_4 \end{bmatrix} \equiv B^T \underline{\tilde{e}}$. Thus $C^T \underline{\tilde{e}} = B^T \underline{\tilde{e}}$ for arbitrary choice of $\underline{\tilde{e}}$, which in turn implies that $B = C$.

We have now shown that a dual-polarized antenna has $B = C$; i.e., the transmit and receive distortion matrices are equal. This result is not generally true if the analysis is extended beyond the antenna to include nonreciprocal elements such as circulators in the receiver chain.

3. THE FLIP MATRIX, STOKES VECTORS, AND THE MUELLER MATRIX

In this section the characterization of scatterers and antennas in terms of complex two-vectors developed in Section 2 will be used to derive simple, explicit definitions for Stokes vectors and backscatter Mueller matrices in terms of Kronecker products. Given these simple expressions, we will quickly derive a number of results frequently stated without proof:

1. The received voltage expressed in terms of Stokes vectors for the receive antenna and backscattered field.
2. The backscattered Stokes vector expressed in terms of the backscatter Mueller matrix.
3. The symmetry of the backscatter Mueller matrix.
4. The "trace rule" for backscatter Mueller matrices.

3.1 The Flip Matrix

We begin with the monostatic convention shown in Figure 2-2, with a right-handed triplet ($\hat{x}_t$, $\hat{y}_t$, $\hat{z}_t$) for the transmit coordinate system and a left-handed triplet ($\hat{x}_s$, $\hat{y}_s$, $\hat{z}_s$) for the scatter coordinate system. It was shown in Section 2 that the backscattered field $\tilde{\underline{e}}$ expressed in the left-handed coordinate system and the received voltage V are given by:

$$\begin{aligned} \tilde{\underline{e}} &= A\underline{t} \\ V &= \underline{r}^T \tilde{\underline{e}} \end{aligned} \tag{3.1}$$

where $\underline{t}$ and $\underline{r}$ are expressed in the right-handed transmit coordinate system, and $\tilde{\underline{e}}$ is expressed in the left-handed backscatter coordinate system (see Figure 3-1). (As in Section 2, the tilde is used to show that a field is expressed in a left-handed system.)

Workers in the field usually use scattered Stokes vectors expressed in a <u>right-handed</u> coordinate system. If one wishes to change the basis of the scattered field to a right-handed system, ($-\hat{x}_s$, $\hat{y}_s$, $\hat{z}_s$) one can flip the x-axis by premultiplying by a flip matrix F:

$$\underline{e} = F\tilde{\underline{e}} \tag{3.2}$$

$$\text{where } F = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.3}$$

Note that while the matrix A is symmetric, the matrix FA is not.

The next section demonstrates how to write Stokes vectors for both transmitted and backscattered fields.

3.2 Stokes Vectors

The transmit and receive Stokes vectors (i.e. the Stokes vectors associated with the fields that would be transmitted by the transmit and receive antennas) are [1]:

$$\underline{s}_t = T(\underline{t} \otimes \underline{t}^*) \tag{3.4}$$

$$\underline{s}_r = T(\underline{r} \otimes \underline{r}^*) \tag{3.5}$$

where

$$T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix} \tag{3.6}$$

and where the symbol $\otimes$ denotes the direct or Kronecker product [5, 6]. We use the *left* Kronecker product: if A and B are two-by-two matrices, then the Kronecker product of A and B is

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix} \tag{3.7}$$

Note that the matrix $\frac{1}{\sqrt{2}}T$ is unitary, since $T^{-1} = \frac{1}{2}T^{\dagger}$.

The scattered Stokes vector (expressed in a left-handed coordinate system) is given by (see the Appendix and [3])

$$\begin{aligned} \tilde{\underline{s}}_e &= T^*(\tilde{\underline{e}} \otimes \tilde{\underline{e}}^*) \\ &= T^*(FF\tilde{\underline{e}}) \otimes (F^*F^*\tilde{\underline{e}}^*) \\ &= T^*(F \otimes F^*)(F\tilde{\underline{e}} \otimes F^*\tilde{\underline{e}}^*) \\ &= T^*(F \otimes F^*)T^{-1}T(\underline{e} \otimes \underline{e}^*) \end{aligned} \tag{3.8}$$

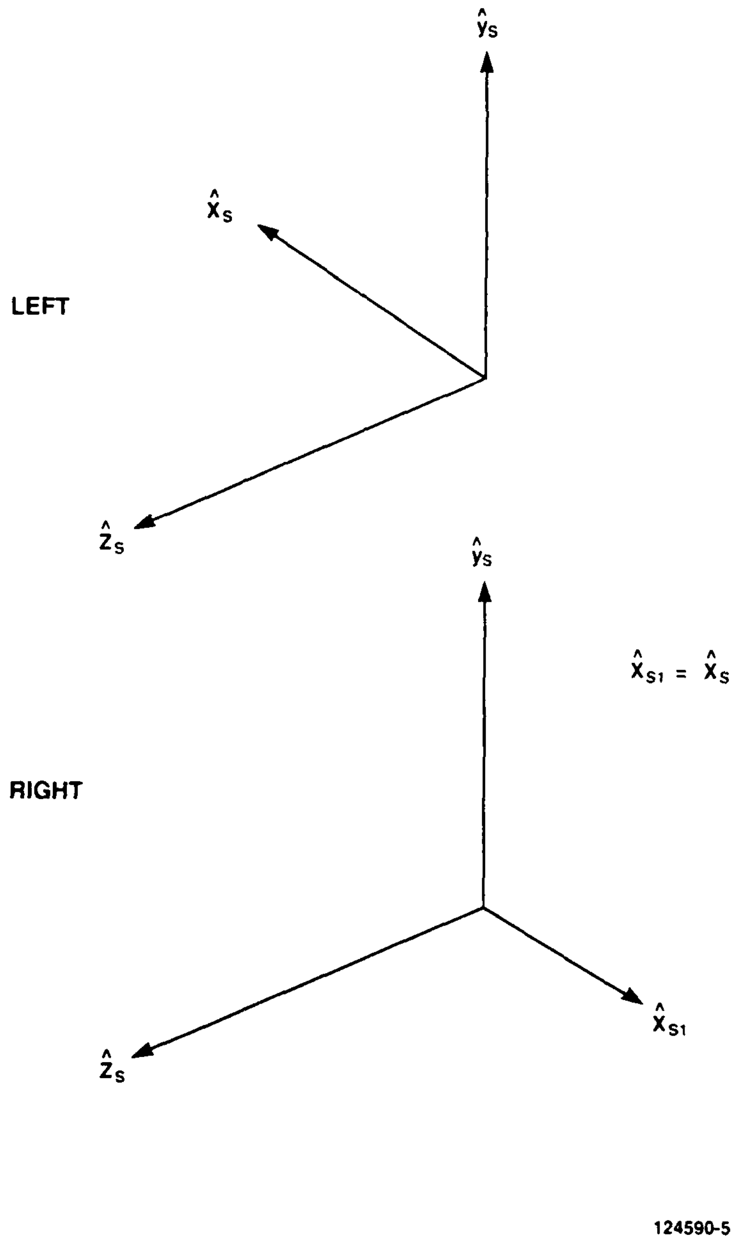

Figure 3-1. Left-handed and right-handed coordinate systems for a backscattered field.

(Here we have used the fact that the matrix product FF is equal to the identity matrix.)

Equation 3.8 can be expressed more compactly by $\underline{\tilde{s}}_e = Z\underline{s}_e$, where $\underline{s}_e = T(\underline{e} \otimes \underline{e}^*)$ is the backscattered Stokes vector expressed in a right-handed coordinate system, and where the 4-by-4 real matrix Z is, as will be shown in Equation 3.12, the Mueller-matrix equivalent of the flip operation:

$$Z \equiv T^*(F \otimes F^*)T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{3.9}$$

3.3 Received Power and The Backscatter Mueller Matrix

Equations 3.4 and 3.8 can be combined to show the relationship between received power, the backscattered Stokes vector, and the backscatter Mueller matrix. The received power is given by

$$\begin{aligned} P = |V|^2 = V \otimes V^* &= (\underline{r}^T\underline{\tilde{e}}) \otimes (\underline{r}^\dagger\underline{\tilde{e}}^*) \\ &= (\underline{r}^T \otimes \underline{r}^\dagger)(\underline{\tilde{e}} \otimes \underline{\tilde{e}}^*) \\ &= (\underline{r} \otimes \underline{r}*)^T(T^*)^{-1}T^*(\underline{\tilde{e}} \otimes \underline{\tilde{e}}^*) \\ &= \frac{1}{2}[(\underline{r} \otimes \underline{r}*)^T T^T]\underline{\tilde{s}}_e \\ &= \frac{1}{2}(T\underline{r} \otimes \underline{r}*)^T\underline{\tilde{s}}_e \\ &= \frac{1}{2}\underline{s}_r^T\underline{\tilde{s}}_e \end{aligned} \tag{3.10}$$

and the scattered Stokes vector is given by

$$\begin{aligned} \underline{\tilde{s}}_e &= T^*\underline{\tilde{e}} \otimes \underline{\tilde{e}}^* \\ &= T^*(A\underline{t} \otimes A^*\underline{t}^*) \\ &= T^*(A \otimes A^*)(\underline{t} \otimes \underline{t}^*) \\ &= (T^*A \otimes A^*T^{-1})(T\underline{t} \otimes \underline{t}^*) \\ &= (\frac{1}{2}T^*A \otimes A^*T^\dagger)\underline{s}_t \end{aligned} \tag{3.11}$$

This latter result can be written as $\tilde{\underline{s}}_e = M\underline{s}_t$, where

$$M = T^*(A \otimes A^*)T^{-1} = \frac{1}{2}T^*(A \otimes A^*)T^\dagger \tag{3.12}$$

is the Mueller matrix relating backscattered to incident Stokes vectors. Note that the scattered Stokes vector is expressed in a left-handed coordinate system, whereas the incident Stokes vector is expressed in a right-handed coordinate system.

Combining Equations 3.10 and 3.12:

$$P = \frac{1}{2}\underline{s}_r^T M \underline{s}_t \tag{3.13}$$

where both the transmit and receive Stokes vectors (i.e., the Stokes vectors corresponding to both antennas, transmit and receive) are expressed in a right-handed coordinate system.

We have developed several tools useful for working with Stokes vectors and Mueller matrices, and have already used these tools to derive results for the scattered Stokes vector (Equation 3.8) and the received power in terms of the Mueller matrix (Equation 3.12). With these tools, it is easy to derive other results that are frequently stated without proof. For example, let us derive the symmetry of the backscatter Mueller matrix and prove the "trace rule".

Symmetry of the Backscatter Mueller Matrix

We can show that the backscatter Mueller matrix M is symmetric, using the symmetry of A and the identities $(M \otimes N)^T = M^T \otimes N^T$ and $(MN)^T = N^T M^T$:

$$\begin{aligned} M^T &= \frac{1}{2}T^*(A \otimes A^*)^T T^\dagger \\ &= \frac{1}{2}T^*(A^T \otimes A^\dagger)T^\dagger \\ &= \frac{1}{2}T^*(A \otimes A^*)T^\dagger \\ &= M \end{aligned} \tag{3.14}$$

Since the backscatter Mueller matrix is real and symmetric, it can have at most ten independent parameters, since the six elements below the main diagonal are equal to their counterparts above the main diagonal. We next show that the number of independent elements is nine or fewer, because the elements on the main diagonal are not independent of each other.

Trace Rule

Let us prove that the (1,1) element of the backscatter Mueller backscatter matrix M is equal to the sum of the other three diagonal elements. This relationship is called the "trace rule" (the trace operator is the sum of the diagonal elements of a matrix) because the relationship can be stated using the trace as follows: $Tr(RM) = 0$, where the matrix R is defined as

$$R \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The proof involves simple substitution and calculation. First the substitution:

$$\begin{aligned} Tr(RM) &= Tr(RT^{*}A \otimes A^{*}T^{-1}) \\ &= Tr(T^{-1}RT^{*}A \otimes A^{*}) \end{aligned} \tag{3.15}$$

where the last step follows from the fact that $Tr(AB) = Tr(BA)$. Next, the value of the matrix $T^{-1}RT^{*}$ can be found by direct calculation:

$$T^{-1}RT^{*} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

This matrix can be represented as a direct (Kronecker) product of a simple 2-by-2 matrix G with its own conjugate:

$$\begin{aligned} T^{-1}RT^{*} &= G \otimes G^{*} \\ \text{where } G &\equiv \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

Thus,

$$\begin{aligned} Tr(RM) &= Tr[(G \otimes G^{*})(A \otimes A^{*})] \\ &= Tr[(GA) \otimes (G^{*}A^{*})] \\ &= Tr(GA)Tr(G^{*}A^{*}) \end{aligned}$$

$$
\begin{aligned}
&= |Tr(GA)|^2 \\
&= |A_{21} - A_{12}|^2 \\
&= 0
\end{aligned}
\tag{3.16}
$$

where the first two steps follow from standard properties of direct products [5]: $(A \otimes B)(C \otimes D) = AC \otimes BD$, and $Tr(A \otimes B) = Tr(A)Tr(B)$, and the last step follows from the symmetry of A.

This completes the proof that the trace is zero.

4. MEASUREMENT OF FORWARD-SCATTER DEPOLARIZATION

The previous sections of this paper described backscatter polarimetry, for which the Mueller matrix reduces to a special form known as the Stokes reflection matrix. This Stokes reflection matrix is symmetric and obeys the "trace rule" (the (1,1) element is equal to the sum of the other three diagonal elements). As a consequence, the matrix has not sixteen but nine independent elements.

In contrast, the Mueller matrix that describes forward scatter, which we refer to here as the transmission Mueller matrix, has all sixteen elements independent.

Analysis of forward scatter is of paramount importance in foliage penetration (FOPEN) systems, in which a radar wave is to pass through tree foliage, bounce off a target hiding under the trees, and pass back up through the foliage. Measurements of foliage depolarization that have been described in the literature do not include a systematic mathematical structure with which to evaluate the measurements. Using the N-port and Stokes-vector theory developed in previous sections of this paper, we will not only describe forward-scatter depolarization, but also develop an algorithm for systematically measuring depolarization using polarimetric active radar calibrators (PARCs).

Section 4.1 describes the problem which is to be solved. Section 4.2 develops the tools needed for describing forward-scatter depolarization: Section 4.2.1 derives the Stokes reflection matrix for a PARC, and Section 4.2.2 demonstrates the equality of the Mueller matrices describing upward-going and downward-going foliage penetration. Section 4.3 presents a new depolarization measurement algorithm, with the relative (depolarization) part in Section 4.3.1 and the absolute (attenuation) part in Section 4.3.2.

4.1 Problem Definition

Assume that the polarization-changing properties of foliage can be summarized by a transmission Mueller matrix N which describes how a Stokes vector $\underline{s}_1$ incident on the foliage is changed to the transmitted Stokes vector $\underline{s}$ which emerges on the other side of the foliage:

$$\underline{s}_2 = N\underline{s}_1 \tag{4.1}$$

Imagine a situation in which a radar wave originates above a treeline in an airborne transmitter, propagates through the trees, bounces off a point target under the trees, and propagates back up through the trees and then back to the radar. Denote the transmission Mueller matrix corresponding to the downward propagation by N_d, the backscatter Mueller matrix (also known as the Stokes reflection matrix) associated with the point target by M, and the transmission

Mueller matrix corresponding to the upward propagation by N_u. Suppose the radar transmits a wave with Stokes vector $\underline{s}_t$. After passing down through the trees, the transmitted Stokes vector is $N_d\underline{s}_t$, and after bouncing off the point target, the scattered Stokes vector is (invoking Equation 3.12) $MN_d\underline{s}_t$. Finally, after passing back up through the foliage, the scattered Stokes vector is $N_uMN_d\underline{s}_t$. If we use the Stokes vector $\underline{s}_r$ to describe the receive antenna, then the received power is

$$\begin{aligned} P &= \frac{1}{2}\underline{s}_r^T(N_uMN_d\underline{s}_t) \\ &= \frac{1}{2}\underline{s}_r^T D\underline{s}_t \\ \text{where } D &\equiv N_uMN_d \end{aligned} \tag{4.2}$$

The objective of the foliage depolarization measurement process is to estimate N_u and N_d from a set of measurements on point targets located under the foliage. To show how this is done, we first develop some mathematical tools, and then we present an algorithm for inferring N_d given Stokes vector measurements of four particular PARCs.

4.2 Mathematical Tools

In this section we will develop some mathematical tools that will be used in Section 4.3 to develop the measurement algorithm. We will derive the Stokes reflection matrix (i.e., the backscatter Mueller matrix) for a PARC, and then will show that the Mueller matrices describing depolarization during downward and upward foliage penetration are related by $N_d = N_u^T$ (so that the foliage depolarization measurement problem reduces to that of finding only one or the other of N_u and N_d).

4.2.1 Stokes Reflection Matrix for a PARC

A PARC is an active device that receives a radar signal with one antenna (usually linearly polarized), amplifies the waveform, and retransmits through a second antenna. The polarization scattering matrix of this device is singular, because the polarization form of the retransmitted field is independent of the incident field. The scattering matrix can thus be represented by the outer product of the complex two-vectors representing the transmit and receive polarizations:

$$A = \bar{\underline{t}}\ \underline{r}^T \tag{4.3}$$

where the tilde indicates a vector expressed in a left-handed coordinate system.

The backscatter Mueller matrix (Stokes reflection matrix) of the PARC can be found using results from Section 3.3 plus the relations $T^{-1} = (1/2)T^{\dagger}$ and $(\underline{a} \otimes \underline{b})^T = \underline{a}^T \otimes \underline{b}^T$:

$$\begin{aligned}
M &= T^{*} A \otimes A^{*} T^{-1} \\
&= T^{*}(\underline{\tilde{t}}\ \underline{r}^T) \otimes (\underline{\tilde{t}}^{*} \underline{r}^{\dagger}) T^{-1} \\
&= T^{*}(\underline{\tilde{t}} \otimes \underline{\tilde{t}}^{*})(\underline{r}^T \otimes \underline{r}^{\dagger}) T^{-1} \\
&= \frac{1}{2}\underline{\tilde{s}}_t(\underline{r}^T \otimes \underline{r}^{\dagger}) T^{\dagger} \\
&= \frac{1}{2}\underline{\tilde{s}}_t(T^{*}\underline{r} \otimes \underline{r}^{*})^T \\
&= \frac{1}{2}\underline{\tilde{s}}_t \underline{\tilde{s}}_r^T
\end{aligned} \tag{4.4}$$

No matter what Stokes vector is received by the PARC, the transmitted Stokes vector will be of the form $\underline{\tilde{s}}_t$. To see this, imagine that Stokes vector $\underline{s}_x$ is incident on the PARC. The scattered Stokes vector is then $M\underline{s}_x = [(1/2)\underline{\tilde{s}}_t\underline{\tilde{s}}_r^T]\underline{s}_x = [(1/2)\underline{\tilde{s}}_r^T\underline{s}_x]\underline{\tilde{s}}_t$, which is a scalar times the vector $\underline{\tilde{s}}_t$.

This result makes sense: the PARC always transmits the polarization $\underline{\tilde{s}}_t$. Note that the Stokes reflection matrix is singular, as is the polarization scattering matrix.

4.2.2 Relationship Between Mueller Matrices for Downward and Upward Foliage Penetration

In this section we will show that $N_d = N_u^T$. This means that characterizing foliage depolarization requires finding only one or the other of these Mueller matrices.

Consider the foliage system shown in Figure 4-1a, with field $\underline{e}_a$ incident from above, field $\underline{e}_b$ transmitted down through the foliage to the area beneath, field $\underline{\tilde{e}}_c$ incident on the same foliage from below after having bounced off a target under the foliage, and field $\underline{\tilde{e}}_d$ transmitted up through the foliage. The latter two fields are expressed in a left-handed coordinate system, consistent with the monostatic convention described in Section 3.

Figure 4-1b shows an equivalent four-port network representation of this system, with Ports 1 and 2 representing the horizontally (H) and vertically (V) polarized electric fields on the top side of the foliage, and Ports 3 and 4 representing the H and V fields on the bottom side of the foliage. The N-port scattering matrix S for this four-port system is a complex, symmetric,

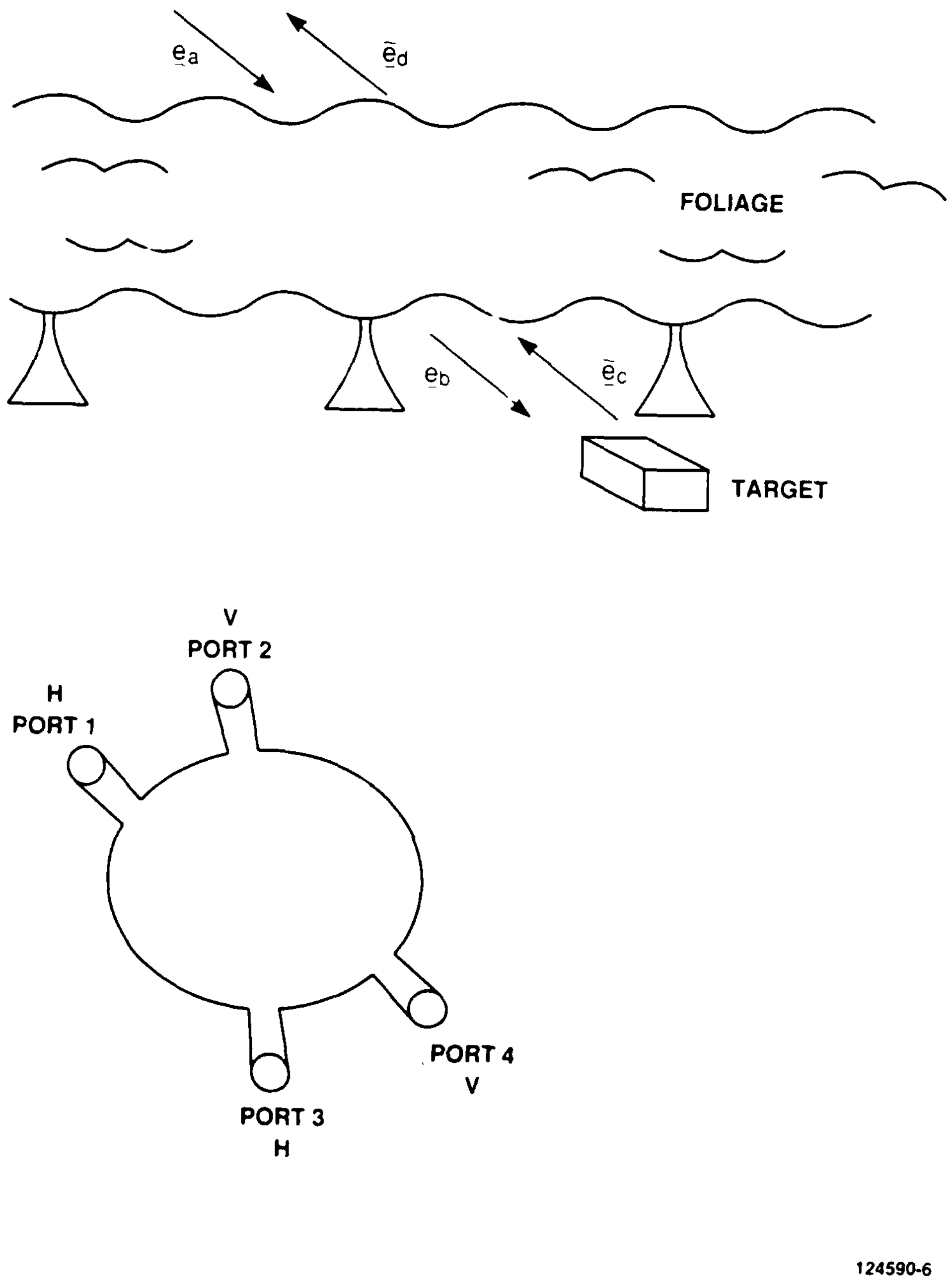

Figure 4-1. Foliage penetration geometry, and equivalent four-port network.

4-by-4 matrix relating scattered to incident waves in the four ports. Let us express S in terms of its two-by-two submatrices, denoted here by the dummy matrices P, Q, and R:

$$S = \begin{bmatrix} P & Q^T \\ Q & R \end{bmatrix} \tag{4.5}$$

The H and V components of the field $\underline{e}_a$ are are incident waves in Ports 1 and 2, while the H and V components of $\underline{e}_b$ are outgoing waves in Ports 3 and 4. The coupling from Ports 1 and 2 to Ports 3 and 4 is described by the lower left submatrix Q of the scattering matrix S:

$$\underline{e}_b = Q\underline{e}_a \tag{4.6}$$

Similarly, the H and V components of the field $\tilde{\underline{e}}_c$ are are incident waves in Ports 3 and 4, while the H and V components of $\tilde{\underline{e}}_d$ are outgoing waves in Ports 1 and 2. The coupling from Ports 3 and 4 to Ports 1 and 2 is described by the upper right submatrix Q^T of the scattering matrix S:

$$\tilde{\underline{e}}_d = Q^T\tilde{\underline{e}}_c \tag{4.7}$$

Now let us repeat the development of Section 3.3, this time working with forward scatter rather than backscatter. We will extend the development by letting the foliage transmission matrix Q be random, and then averaging over the ensemble of all the random matrices Q in the probability space (this ensemble average is denoted by an overbar). The downward-transmitted Stokes vector is

$$\begin{aligned} \underline{s}_b &= T\overline{\underline{e}_b \otimes \underline{e}_b^*} \\ &= T\overline{Q\underline{e}_a \otimes Q^*\underline{e}_a^*} \\ &= T\overline{(Q \otimes Q^*)(\underline{e}_a \otimes \underline{e}_a^*)} \\ &= T\overline{Q \otimes Q^*}\ \overline{\underline{e}_a \otimes \underline{e}_a^*} \\ &= (T\overline{Q \otimes Q^*}T^{-1})(T\overline{\underline{e}_a \otimes \underline{e}_a^*}) \\ &= (\frac{1}{2}T\overline{Q \otimes Q^*}T^{\dagger})\underline{s}_a \end{aligned} \tag{4.8}$$

This result can be written $\underline{s}_b = N_d\underline{s}_a$, where

$$N_d = T\overline{Q \otimes Q^*}T^{-1} = \frac{1}{2}T\overline{Q \otimes Q^*}T^\dagger \tag{4.9}$$

is the Mueller matrix relating downward-transmitted to incident Stokes vectors. Note that both the incident and the transmitted Stokes vectors are expressed in a right-handed coordinate system.

Similarly, the incident and transmitted Stokes vectors for the upward-traveling waves ($\underline{\bar{s}}_c$ and $\underline{\bar{s}}_d$) are related (as expressed in a left-handed coordinate system) by

$$\begin{aligned}
\underline{\bar{s}}_d &= T^*\overline{\underline{\bar{e}}_d \otimes \underline{\bar{e}}_d^*} \\
&= T^*\overline{Q^T\underline{\bar{e}}_c \otimes Q^\dagger\underline{\bar{e}}_c^*} \\
&= T^*\overline{(Q^T \otimes Q^\dagger)(\underline{\bar{e}}_c \otimes \underline{\bar{e}}_c^*)} \\
&= T^*\overline{Q^T \otimes Q^\dagger}\ \overline{\underline{\bar{e}}_c \otimes \underline{\bar{e}}_c^*} \\
&= (T^*\overline{Q^T \otimes Q^\dagger}(T^*)^{-1})(T^*\overline{\underline{\bar{e}}_c \otimes \underline{\bar{e}}_c^*} \\
&= (\frac{1}{2}T^*\overline{Q^T \otimes Q^\dagger}T^T)\underline{\bar{s}}_c
\end{aligned} \tag{4.10}$$

This result can be written $\underline{\bar{s}}_d = N_u\underline{\bar{s}}_c$, where

$$N_u = T^*\overline{Q^T \otimes Q^\dagger}(T^*)^{-1} = \frac{1}{2}T^*\overline{Q^T \otimes Q^\dagger}T^T \tag{4.11}$$

is the Mueller matrix relating transmitted to incident Stokes vectors for the upward-traveling wave.

The matrices N_u and N_d can now be related to one another by direct calculation:

$$\begin{aligned}
N_u^T &= \frac{1}{2}T\overline{(Q^T \otimes Q^\dagger)^T}T^\dagger \\
&= T\overline{Q \otimes Q^*}(\frac{1}{2}T^\dagger) \\
&= T^*\overline{Q \otimes Q^*}T^{-1} \\
&= N_d
\end{aligned} \tag{4.12}$$

We now have N_d in terms of N_u. Combining Equations 4.2 and 4.12, the net backscatter Mueller matrix for the foliage/target/foliage combination is

$$\begin{aligned} D &= N_u M N_d \\ &= N_d^T M N_d \\ &= N_u M N_u^T \end{aligned} \tag{4.13}$$

The results shown in Equations 4.4, 4.10, and 4.13 will be used in Section 4.3.

4.3 Measuring Foliage Depolarization using PARCs

In order to use the results derived in Section 4.2, let us suppose that a radar and four PARCs are set up as follows:

1. The airborne radar transmits a single, fixed polarization (say, vertical polarization). Let this polarization correspond to Stokes vector $\underline{s}_0$. This Stokes vector is *not* normalized, i.e., it has a first element which is not equal to unity, but rather depends on the (known) radar tranmsit power.
2. The radar illuminates four PARCs that have been placed under foliage.
3. The PARCs are built so that their receivers are all matched to the radar transmit polarization $\underline{s}_0$. The PARC receiver gain will be described by the constant of proportionality β, so that the receive Stokes vector common to all of the PARCs is $\beta\underline{s}_0$.
4. The PARCs have four different transmit polarizations, identified by Stokes vectors $\underline{x}_i, i = 1, ..., 4$.

With these assumptions, the net backscatter Mueller matrix for the foliage/PARC/foliage is (for the ith PARC):

$$D_i = \frac{1}{2} N_u \underline{\tilde{x}}_i (\beta \underline{\tilde{s}}_0)^T N_u^T \tag{4.14}$$

and the backscattered Stokes vector from the ith PARC, which we denote by $\underline{\tilde{f}}_i$, is

$$\begin{aligned}
\underline{\tilde{f}}_i &= D_i \underline{s}_0 \\
&= \frac{1}{2} N_u \underline{\tilde{x}}_i (\beta \underline{\tilde{s}}_0)^T N_u^T \underline{s}_0 \\
&= \alpha N_u \underline{\tilde{x}}_i \qquad (4.15)
\end{aligned}$$

where $\alpha \equiv \frac{1}{2}(\beta \underline{\tilde{s}}_0)^T N_u^T \underline{s}_0$ is an unknown scalar multiplier that depends on attenuation through the foliage to the PARC receive antenna.

The estimation of N_u will be achieved in two steps: first, the estimation of a normalized N_u (Section 4.3.1), and then the estimation of the absolute attenuation corresponding to α (Section 4.3.2). Since the PARC receive antennas are all matched to the radar transmit antenna, enough power should be available to the PARCs to result in a signal-to-noise ratio adequate for parameter estimation.

4.3.1 Measurement of Relative Depolarization

Equation 4.15 is really four equations, one for each PARC. Let us combine them into one matrix equation. To do this, arrange the vectors $\underline{\tilde{f}}_i$ to form matrix F, and also arrange the vectors $\underline{\tilde{x}}_i$ to form matrix X. Next, define the matrix $N \equiv \alpha N_u$. Then the four equations can be expressed compactly as

$$F = NX \qquad (4.16)$$

The matrices F and X are both known: the X matrix simply expresses our specifications for the four PARCs, and the F matrix is measured directly. Thus, we can solve for the desired matrix N:

$$N = FX^{-1} \qquad (4.17)$$

The unknown matrix N_u is then proportional to N. In section 4.3.2, we will discuss how to determine the constant of proportionality α.

Strictly speaking, in order for this scheme to work, all we need is any set of four linearly independent $\underline{\tilde{x}}_i$ that span the space of real four-vectors and that therefore form an invertible matrix X. However, to ensure that the matrix X is well-conditioned (and therefore has an inverse which is robust in the presence of noise), we should pick four $\underline{\tilde{x}}_i$ that are as different from each other as possible. A good set would be H, V, 45 degree linear polarization, and left-circular polarization, with Stokes vectors that can be found from [3] as:

$$\tilde{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \tilde{x}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \tilde{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \tilde{x}_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{4.18}$$

A word on the choice of left-circular polarization is appropriate here. It is necessary to put some energy into the last element $(\tilde{x}_4)_4$ of the fourth transmit Stokes vector $\tilde{x}_4$ in Equation 4.18, in order to stimulate the last column of N_u in Equation 4.15. PARCs that transmit linear polarization (for example, $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{x}_3$ in Equation 4.18) have no energy in the last element of their Stokes vectors, and are thus unable to stimulate the last column of N_u.

While it may be difficult to construct a circularly polarized patch antenna such as that needed to transmit Stokes vector $\tilde{x}_4$, it is not impossible [7], and at least one PARC vendor has expressed a willingness to build a PARC that uses this type of antenna.

This choice of PARCs gives us an X matrix

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with an inverse

$$X^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

To summarize, the depolarization measurement approach proposed here involves the use of four PARCs, all of which are designed to receive vertical polarization but which transmit H, V, 45 degree linear, and left-circular, respectively. The first three are standard-issue PARCs (VH, VV, V/45-degree), while the last (V/left-circular) may involve some effort in modifying or redesigning an existing PARC.

4.3.2 Absolute (Radiometric) Measurement

In the preceding section, we showed how to estimate the one-way Mueller matrix of foliage up to but not including an arbitrary scalar multiplier α: after performing this partial calibration, we know αN_u but not N_u. To determine α (and therefore N_u), the following steps would be taken:

1. Measure the Mueller matrix D_1 for the VV PARC.
2. Remove the effect of relative depolarization by pre- and post-multiplying D_1 by the inverses of the normalized transmission Mueller matrices determined as shown in Section 4.3.1.
3. Complete the determination of α by using the known magnitude of the VV PARC response.

The Mueller matrix for the VV PARC is

$$D_1 = \frac{1}{2} N_u \bar{\mathbf{x}}_1 (\beta \bar{\mathbf{s}}_0)^T N_u^T \tag{4.19}$$

We adjust D_1 to remove depolarization effects by premultiplying D_1 by the inverse of $N = \alpha N_u$ and postmultiplying by the inverse of N^T. The result is the relative-polarimetric-adjusted version $\hat{D}_1$:

$$\begin{aligned} \hat{D}_1 &\equiv (\alpha N_u)^{-1} D_1 (\alpha N_u^T)^{-1} \\ &= \frac{1}{2\alpha^2} \bar{\mathbf{x}}_1 (\beta \bar{\mathbf{s}}_0)^T \end{aligned} \tag{4.20}$$

But we know the values of the vectors on the right side (they are the transmit and receive Stokes vectors for the VV PARC, which is itself calibrated), and we can measure the matrix on the left side; thus we can solve for the attenuation α. Let the first elements of the vectors on the right hand side be $(\bar{\mathbf{x}}_1)_1$ and $(\beta \bar{\mathbf{s}}_0^T)_1$, respectively, and the upper left element of the matrix D_1 be $(D_1)_{11}$. Then

$$\begin{aligned} (\hat{D}_1)_{11} &= \frac{1}{2\alpha^2} (\bar{\mathbf{x}}_1)_1 (\beta \bar{\mathbf{s}}_0^T)_1 \\ \Longrightarrow \alpha &= \sqrt{\frac{(\bar{\mathbf{x}}_1)_1 (\beta \bar{\mathbf{s}}_0^T)_1}{2(\hat{D}_1)_{11}}} \end{aligned} \tag{4.21}$$

We have now completed the measurement of the upward-traveling Mueller matrix, since we know αN_u and α, and can therefore solve for N_u. As was shown in Section 4.2.2, this also gives us N_d.

The measurement of depolarization during foliage penetration is now complete, since both the upward-traveling and downward-traveling Mueller matrices have been found.

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APPENDIX A
STOKES VECTORS FOR RIGHT- AND LEFT-HANDED SYSTEMS

The Stokes vector $\underline{s}$ is defined in [3] in terms of e_x and e_y, the elements of the complex two-vector describing the electric field (expressed in a right-handed coordinate system) by:

$$\underline{s} = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} \equiv \begin{bmatrix} |e_x|^2 + |e_y|^2 \\ |e_x|^2 - |e_y|^2 \\ 2\mathrm{Re}[e_x e_y^*] \\ -2\mathrm{Im}[e_x e_y^*] \end{bmatrix} \tag{A.1}$$

Section 8.2 of [3] (using an idea from [1]) shows how this expression can be more elegantly expressed using a Kronecker product:

$$\begin{aligned} \underline{s} &= T(\underline{E} \otimes \underline{E}^*) \\ \text{where } \underline{E} &= \begin{bmatrix} e_x \\ e_y \end{bmatrix} \\ \text{and where } T &\equiv \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix} \end{aligned} \tag{A.2}$$

Section 8.3 of [3] shows that, in a left-handed system, the definition of s_3 must be changed from $-\mathrm{Im}[e_x e_y^*]$ to $+\mathrm{Im}[e_x e_y^*]$ because, for example, the Stokes vector for left-circular polarization must be $\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}^T$ regardless of whether the complex two-vector describing the left-circular field is expressed in a right-handed or left-handed coordinate system (the representations being $\frac{1}{\sqrt{2}}[\ 1 \quad j\ ]^T$ or $\frac{1}{\sqrt{2}}[\ 1 \quad -j\ ]^T$, respectively). As a result of this re-definition of s_3, one must use the conjugate of T instead of T when expressing a Stokes vector in a left-handed coordinate system:

$$\begin{aligned} \tilde{\underline{s}} &= T^*(\tilde{\underline{E}} \otimes \tilde{\underline{E}}^*) \\ \text{where } \tilde{\underline{E}} &= \begin{bmatrix} e_x \\ e_y \end{bmatrix} \end{aligned} \tag{A.3}$$

and where T^* is the conjugate of the matrix T defined in Equation A.2.

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This tutorial paper on polarimetric definitions uses N-port network theory to prove results that are frequently stated without proof in the literature, including the symmetry of the polarization scattering matrix in backscattering, the equation for received voltage, and the equations defining Stokes vectors and backscatter Mueller matrices. Appropriate N-port networks are defined for a single-polarized antenna, a dual-polarized antenna, and a backscattering target. An important result is demonstrated: reciprocity (symmetry of the polarization scattering matrix) is meaningful *only* in the context of the monostatic convention, for which the coordinate system is the same for both transmit and receive. This, in turn, implies a change in the handedness of the coordinate system, so that scattered fields and Stokes vectors are expressed in opposite handedness from incident fields and Stokes vectors.

These results are used as tools in the derivation of a new technique for measuring depolarization in foliage penetration (FOPEN). Four polarimetric active radar calibrators (PARCs) are used with an algorithm to completely measure the Mueller matrices describing downward-going and upward-going foliage penetration.

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